Finite-Element and Boundary-Element Analysis of Craze Micromechanics

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Synopsis

The two numerical methods are used to estimate craze surface displacements and stresses for both isolated crazes and crazes at crack tips. The results are compared with the predictions of craze micromechanics models. The investigation includes the computation of the craze surface stress profile required to maintain a given craze opening displacement profile. The boundary element program requires less computer time than the finite element one, and similar results are obtained. The analysis also considers the craze surface displacement profile corresponding to an assumed craze surface stress distribution. The element methods produce results which are approximately the same as those obtained using the model of Verheulpen–Heymans and Bauwens.

INTRODUCTION

Craze micrimechanics models consider the displacement and stress profiles of a craze normal to its surface, i.e., the interface of the craze and the polymeric matrix. In many of the models, one of the profiles is assumed, and the other is estimated. It is not necessary to assume either when using a craze model[1] which has previously been applied to the computation of craze displacements and stresses.[2] The craze was modelled by a slit with springs, representing craze fibrils, applied at nodal points. The finite element package included linear springs, and the computed profiles illustrated the limitation of the implicit assumption of linear-elastic craze fibril behavior. It was not possible to model fibrils more accurately using nonlinear springs and dashpots. However, a craze surface stress profile which was consistent with that computed using a Fourier transform procedure was obtained when experimental displacements of an isolated craze were substituted for spring displacements. The present study extends this work to consider crazes at crack tips and to determine the craze opening displacement profile produced by a given craze surface stress distribution. The boundary element method[3] is easy to input and is economical in computer time. It is suitable for the present application, which is essentially a boundary problem.

CRAZE MICROMECHANICS MODELS

These models have been reviewed by Kramer,[4] and in this section the main emphasis is on models which are relevant to the present investigation.

The first attempt to estimate from craze displacements was made by Knight,[5] who applied the Fourier transform analysis of Sneddon[6] to the problem. Knight assumed that the surface displacement was constant for most of the craze length

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and that it reduced to zero at the tip. The craze surface stress profile was unrealistic and was qualitatively similar to ones obtained[1] assuming Hookean behavior of the fibrils. The spectral method used by Knight has recently[7,8] been applied to experimental displacement profiles.

The stress $\sigma_s(x)$ normal to the craze surface is estimated by superposing the uniform applied stress σ_a on the stresses $\Delta S(x)$ required to maintain the profile of craze surface displacement $v(x)$ in the absence of applied stress. The Fourier transform procedure for evaluating $\Delta S(x)$ is convenient using a computer program, but the relationship between the craze opening displacement and surface stress profiles is shown more clearly by reference to an alternative solution[4] to the governing equation. The distributed dislocation method is used to determine the stress at x due to dislocations between the points x^1 and $x^1 + dx^1$, and it is found that $\Delta S(x)$ is given by the equation

$$\Delta S(x) = -\frac{E^*}{2\pi}\int_{-a}^{a}\frac{v^1(x^1)dx^1}{x - x^1} \tag{1}$$

where v^1 is the displacement derivative. E^* is equal to E for plane stress and $E/(1 - \nu^2)$ for plane strain. The dislocation method may be used in this case, even though the displacements are produced by craze fibrils, since the calculated stresses arise from elastic deformation of the matrix at the interface.

Several craze models assume a two-stage craze surface stress distribution. In this paper computed displacements are compared with those predicted by Verheulpen-Heymans and Bauwens.[9] Their model was derived for the case of a single craze of half-length a in an infinite sheet, which was subjected to a uniform applied tensile stress σ_a. The stress normal to the craze boundry is σ_c in the body of the craze and σ_t in the tip zone, as shown in Figure 1. The length r of the zone is such that there is no stress singularity at the tip. This condition is satisfied when

$$\frac{\sigma_a - \sigma_c}{\sigma_t - \sigma_c} = \frac{\cos^{-1}(1 - r/a)}{\pi/2} \tag{2}$$

A similar stress distribution is assumed by Chudnovsky, Palley, and Baer[10] in their thermodynamic approach to quasiequilibrial craze growth. The base and the tip zone correspond to the inert and active zones, respectively, in their model. The figure can also represent other models if a is assumed to be the half-length of a discontinuity. In the model of Argon and Salama[11] the craze length is equal to $a - r$ and the constant stresses are σ_c in the craze and σ_t in the plastic zone at the craze tip. The Dugdale model[12] has also been applied to crazing,[13] and in this case the crack length is $a - r$, σ_c is zero, and σ_t is the stress in the line yield zone, i.e., the craze.

Verheulpen-Heymans and Bauwens[9] used the Muskhelishvili[14] conformal mapping method to derive general expressions for stresses and displacements. The equations were printed incorrectly in the original paper and the amended[15] equation for the surface displacement of the craze boundary, where $x \leq a - r$, $y = 0$, is

$$2GV = \frac{\sigma_t - \sigma_c}{2\pi}(k + 1)\left\{x\ln\frac{(a - r)(a^2 - x^2)^{1/2} - x[r(2a - r)]^{1/2}}{(a - r)(a^2 - x^2)^{1/2} + x[r(2a - r)]^{1/2}} + (a - r)\ln\frac{(a^2 - x^2)^{1/2} + [r(2a - r)]^{1/2}}{(a^2 - x^2)^{1/2} - [r(2a - r)]^{1/2}}\right\} \tag{3}$$

IDEALIZATION OF THE PROBLEM

The numerical analysis was applied to the case of a central discontinuity in a sheet of a glassy polymer which was subjected to applied uniaxial tension. The finite element and boundary element idealizations are shown in Figure 2, in which the craze tip mesh is drawn to an enlarged scale.

In the craze tip region in the midnodes of the boundary elements coincide with the finite element nodes. Due to symmetry, one quadrant only was considered in the finite element program and in the initial part of the boundary element analysis. However, the boundary element results were inaccurate at the corner corresponding to the craze base. This difficulty can be overcome by refining

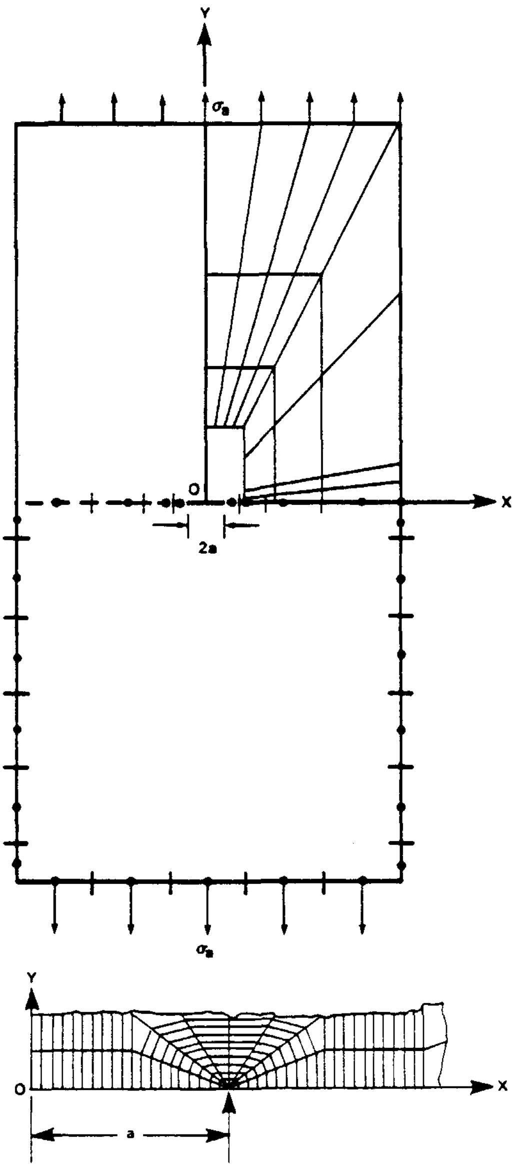


Fig. 2. Finite and boundary element idealization with the craze tip mesh shown to an enlarged scale.

the mesh or by considering the whole plate with two separate boundaries, viz., the sheet boundary and the craze surface. In this case an alternative approach was adopted, and a half-plate mesh was used since the craze surface is then at an appreciable distance from the corner of the mesh. The ratio of plate width to discontinuity length was such that the results approached those of an infinite plate.

RESULTS

In order to check the suitability of the boundary element method for craze micromechanics analysis, it was first applied to a case which had been analyzed using the finite element method.[2] Lauterwasser and Kramer[7] measured the surface displacements of an isolated craze, and their results were included in the input to the boundary element program which is based on constant elements. The boundary element and finite element estimates of the craze surface stress profile are plotted in Figure 3. There is good agreement between the two estimates and the Fourier transform analysis results, which are also plotted.

The next stage in the analysis was to calculate craze surface displacements corresponding to a given craze surface stress distribution. The assumed distribution, for an isolated craze of half-length 1 mm, is shown in Figure 4(a). The surface tractions were specified in the inputs to the programs. The corresponding craze surface displacement profiles are plotted in Figure 4(b), and it is seen that the numerical methods give displacement profiles which approximate that of the model.

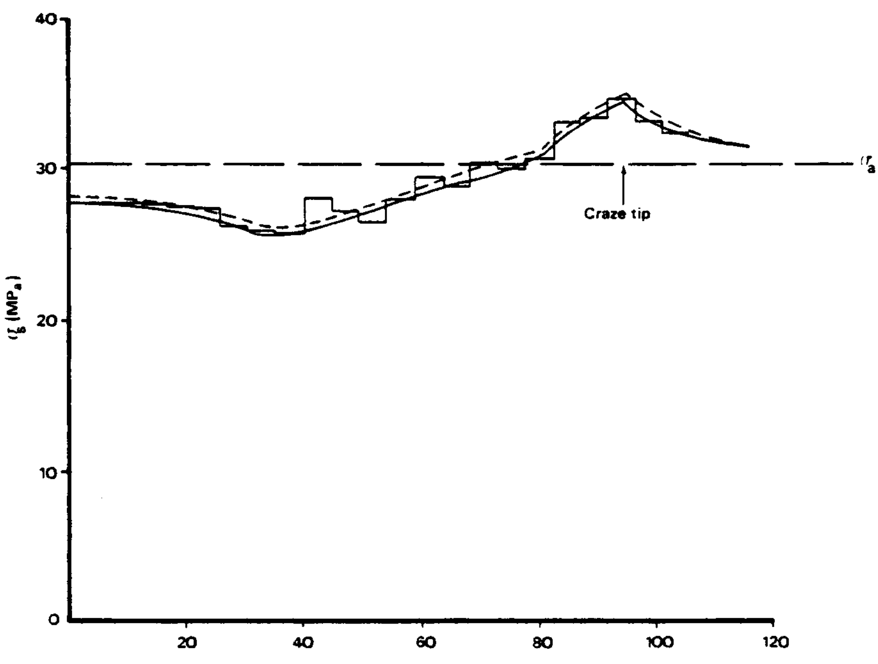


Fig. 3. Craze surface stress profiles obtained by the Fourier transform procedure (after Ref. 7) and the element methods: (- - -) Fourier transform; (—) finite element; (⌐L_) boundary element.

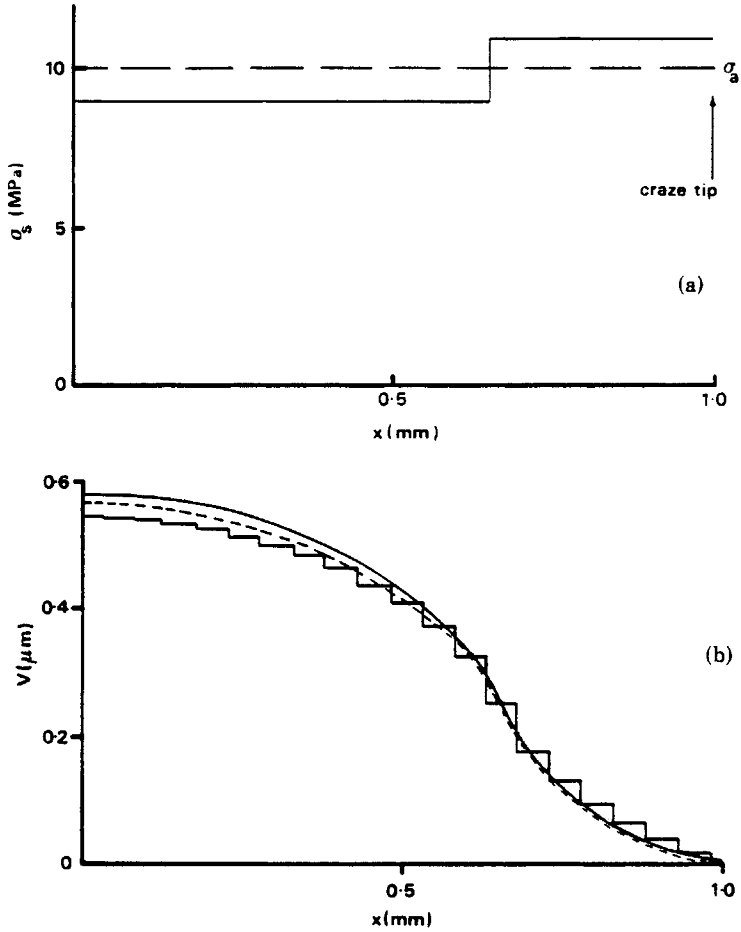


Fig. 4. (a) Craze surface stress distribution for a single craze; (b) craze opening displacement profile corresponding to (a): (—) model; (---) finite element; (⌐L) boundary element.

The case of the craze at a crack tip was approached by putting σ_c equal to zero, producing the stress distribution of Figure 5(a), and then using eq. (2) to calculate the tip stress σ_t corresponding to a given r/a. The craze length was much greater than the length of the crack. This is typical of crazes grown from a starter crack in polystyrene. The craze surface displacement profiles are shown in Figure 5(b), and the trend is similar to that of the isolated craze. Computed results are again in satisfactory agreement with the model. The finite element method gives an estimate of the crack tip opening displacement which differs from the theoretical value by less than 4%, but the method is slightly less accurate in its estimate of displacements elsewhere on the crack surface. The error in these nodal displacements varies between 5% and 6%. There are few nodes on the crack surface, and the accuracy could be improved by refining the mesh in this region. When the crack length is much greater than the craze length, the mesh produces inaccuracies in craze opening displacements since there are few craze tip nodes in this case. The assumed stress distribution is similar to that of the Dugdale model and results which are in good agreement with the model have been obtained both by the finite element[17] and the boundary element[18] methods.

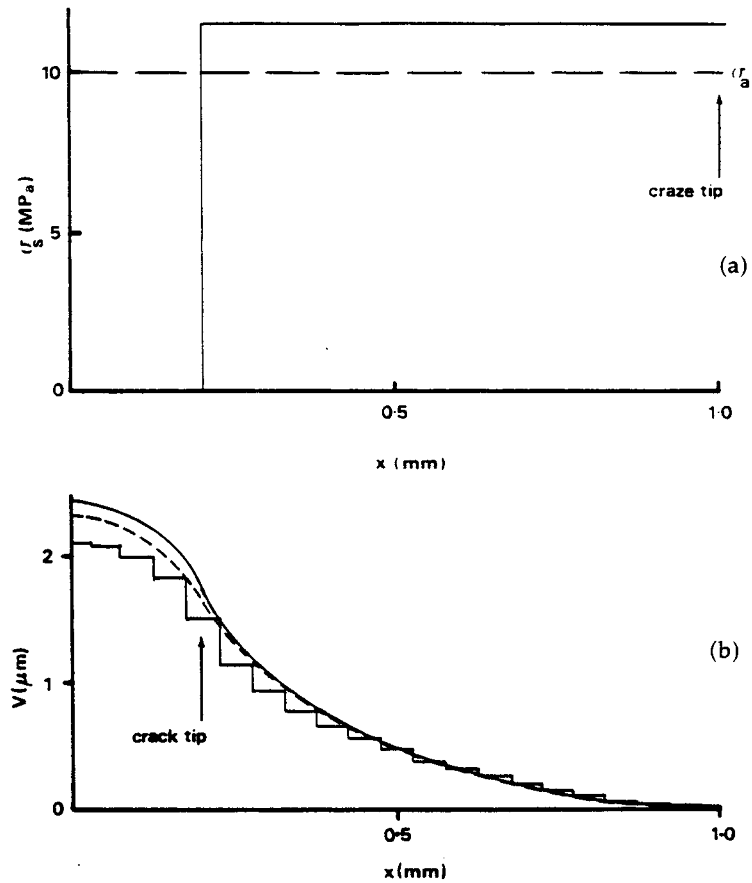
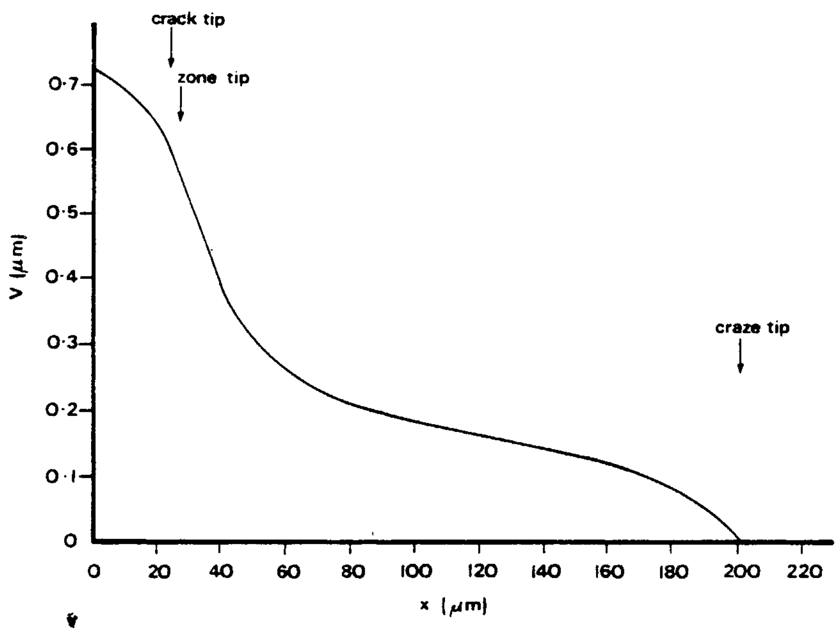
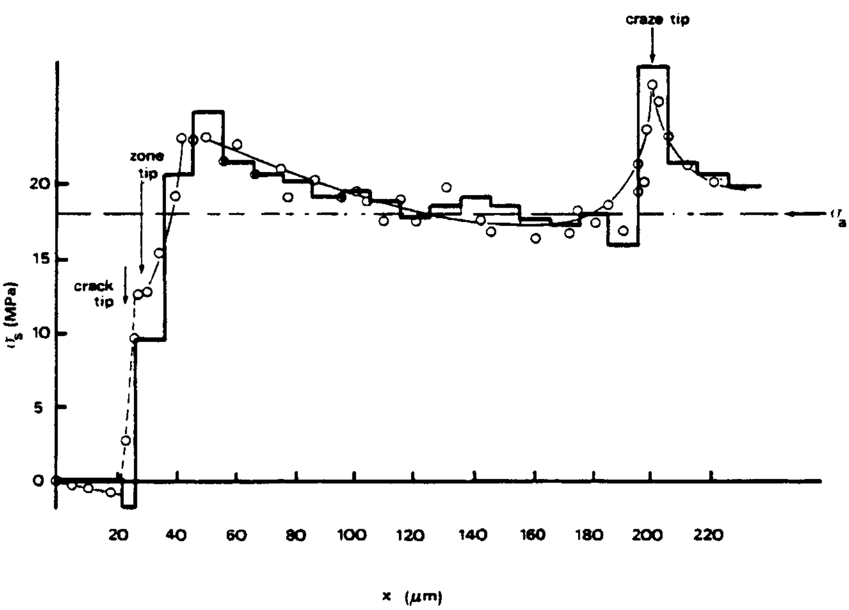


Fig. 5. (a) Surface stress profile for a craze at a crack tip; (b) surface displacement profile produced by (a): (—) model; (- - -) finite element; (⌐L_) boundary elements.

The numerical methods have been used with idealized boundary conditions, and it is desirable to determine the effect of imposing more realistic conditions. This has been accomplished using some of the data of Chan, Donald, and Kramer[8] for crazes at crack tips in thin polystyrene films. Figure 6(a) shows the displacement profile in which the measured surface displacements of the craze and a small deformation zone at the crack tip are joined by a smooth curve to crack opening displacements calculated using the Dugdale model. The Fourier transform procedure used for estimating $\Delta S(x)$ calculates stresses due to displacements of a slit in an infinite plate whereas the plate obviously has finite width and the "crack" is a diamond-shaped indentation. The element methods were used to determine crack surface displacements and craze surface stresses for both idealized and actual boundary conditions. The results were approximately the same in all four cases. A typical profile is shown in Figure 6(b), where comparison is made of the Fourier transform results and the surface stress profile obtained using the boundary element method, taking into account the shape of the crack and the width of the plate. The computed crack surface displacements are within 6% of the calculated values. The boundary element profile for an infinite plate differs by 2% from that shown for the finite width plate.



(a)



(b)

Fig. 6. (a) Surface displacement profile for a craze at the tip of a crack (after Ref. 8); (b) Comparison of the boundary element and Fourier transform estimates of the surface stress distribution required to maintain (a): (O) Fourier transform; (⌐L) boundary element.

DISCUSSION

The similarity between the results of the spectral method and the two element methods is predictable. The relationship between the finite element and Fourier methods was studied by Holmes.[19] It was shown, for a regular mesh construction, that it is possible to recast the finite element method into a Fourier method provided that the frequencies are not required to be evenly spaced. Boundary element results are generally in good agreement with finite element ones, the difference in values depending on the mesh construction and the type of element used.

The computer time required by the boundary element method is usually less than that needed in order to analyze the same problem by finite elements. In the present investigation, the finite element program to compute stresses from prescribed displacements requires four times as much computer time as the boundary element program. When displacements are computed from stresses, the boundary element time is the same, but the finite element time is reduced. The finite element time include the overheads of the package while the boundary element times relate to a two-dimensional elasticity program with constant elements. These elements require less computer time than linear and higher order ones, which produce more accurate results. Constant elements produce satisfactory results in this case, and the use of higher order elements would not be justified.

The results of the numerical analysis show that computed surface displacements are consistent with the assumed surface distribution, although it does not necessarily follow that the assumption is correct. In fact, it has been suggested[20] that the Dugdale model is not fully adequate to describe craze profiles. Nevertheless, computed surface stress profiles corresponding to measured displacement profiles indicate that the two-stage stress distribution is a reasonable approximation both for isolated crazes at crazes at crack tip. In the latter case a modified Dugdale model, with a two-stage craze stress distribution, is more appropriate.

Having established that the two numerical methods give results which approximate to the predicted ones when the assumptions and boundary conditions of the models apply, it will be possible to extend the analysis when further craze micromechanics data is available. The geometry of the plate and the crack can be considered, and the boundary conditions of the experiment can be included in the input to the program. The models considered assume a linear-elastic matrix, but it is not necessary to impose this restriction when using the two numerical methods.

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